DiMeo Schneider & Associates, L.L.C.

White Paper

DiMeo Schneider & Associates, L.L.C.
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# The Low Volatility Dividend

*Diversification can increase returns; not just lower risk*

Diversification not only reduces risk, but can be an important tool for increasing a portfolio's expected long-term return. Reducing a portfolio's volatility by adding a low-correlating investment can markedly improve a portfolio's expected long-term return, even if the new investment does not have a higher expected return than the investment it replaced. How is it possible for a portfolio to have a higher expected return than the sum of its parts? The first step in understanding the **Low Volatility Dividend** is to differentiate *Arithmetic* and *Geometric* investment returns. Perhaps the best way to conceptualize the difference is through a simple example where a $100 portfolio loses 50% in year 1, then gains 50% in year 2. The average *arithmetic* return is 0%, calculated simply by averaging -50% and +50%. However, the $100 turned into $75 over the two-year period. Losing 50% in year 1 takes the $100 portfolio down to $50. Gaining 50% in year 2 brings the $50 portfolio up to $75. So while the average *arithmetic* return was 0%, the portfolio lost $25 (or -13.4% compounded per year). Therefore, the average annual *geometric* return is -13.4%.

**Exhibit I: Arithmetic vs. Geometric Returns:**
Example:
Year 1 Return = -50%
Year 2 Return = 50%

*Arithmetic Annual Return* is calculated by summing the annual returns and dividing by the number of years.
**Arithmetic annual return = (-50% + 50%)/2 = 0%**

*Geometric Annual Return* is calculated by taking the product of [annual return + 1] to the $n^{th}$ root (n = number of years).
***Geometric annual return*** = $[(1 + -0.50)*(1 + 0.50)]^{1/2} - 1 = \underline{-13.4\%}$

The *arithmetic* and *geometric* returns are only equal when a portfolio has the same return every year (or no volatility of returns). Otherwise, the *geometric* return for any time period is always less than the *arithmetic* return. And the higher the volatility, the greater the difference between a portfolio's *arithmetic* and *geometric* annual returns. Simply put, investment losses drain a portfolio's long-term return more than investment gains of the same percentage boost it.

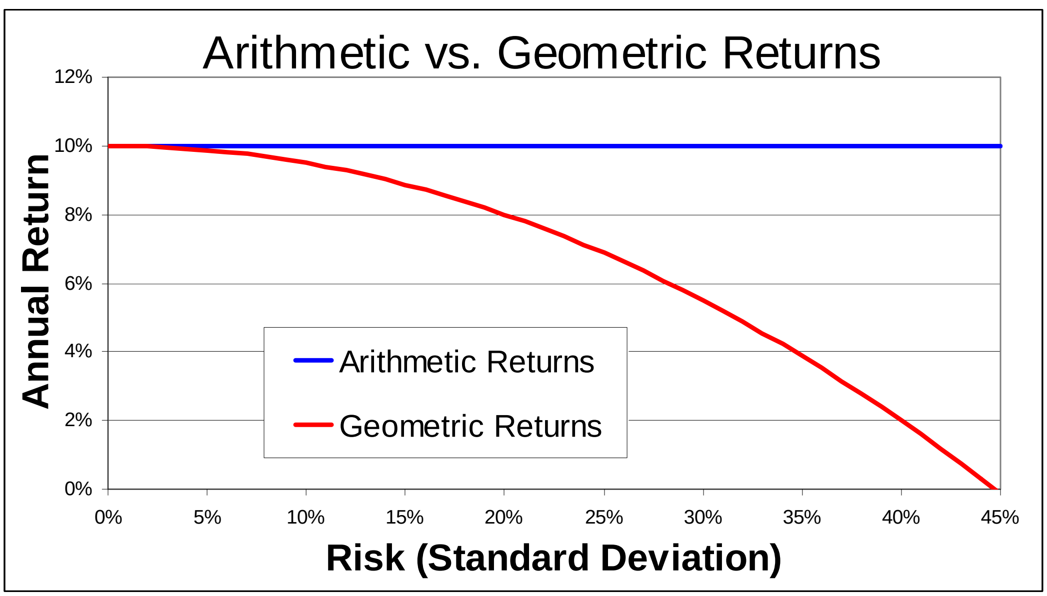
Mathematically speaking, the *geometric* return of a portfolio can be estimated by subtracting [$(\text{standard deviation})^2/2$] from the *arithmetic* return. Hypothetically speaking, if US stocks are expected to generate a 10% annual *arithmetic* return with a 17% annual standard deviation, the annual *geometric* return is expected to be 8.6% [$10\% - (0.17)^2/2$].

**Therefore, any diversifying action that reduces a portfolio's volatility, but maintains the same expected *arithmetic* return, will increase a portfolio's *geometric* returns.** Since *geometric* returns reflect economic reality, maximizing time-horizon *geometric* returns should be the goal. *Arithmetic* returns are irrelevant.

*A 10% arithmetic and 8.6% geometric return forecast for US Stocks are for illustrative purposes only. There is no guarantee of any return.*

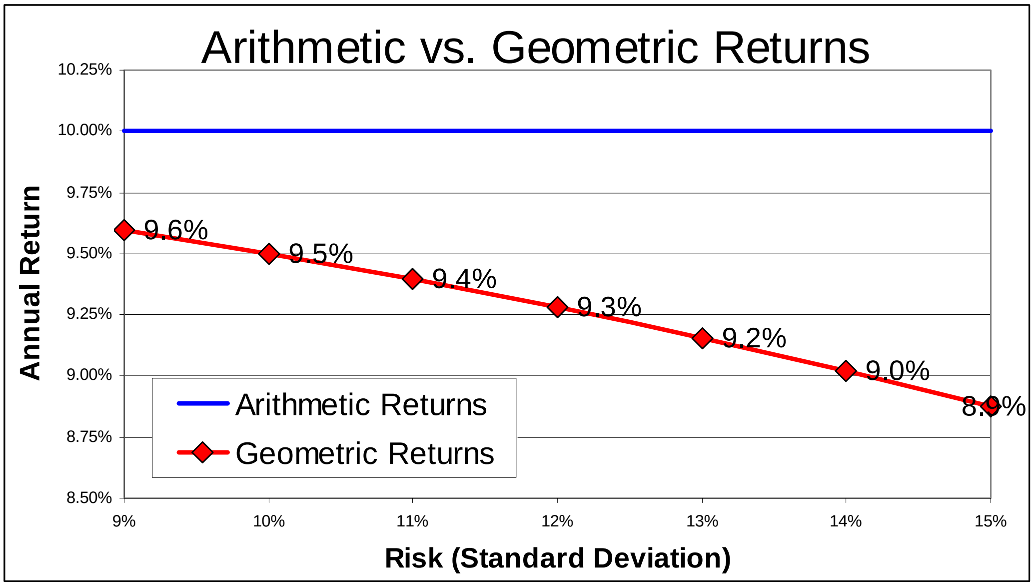
Exhibit II compares the relationship between arithmetic returns, geometric returns and volatility. The analysis shown graphically in Exhibit II makes the assumption that a portfolio has a 10% *arithmetic* return. It also illustrates the relationship between *arithmetic* and *geometric* returns based on volatilities (or standard deviations) ranging from of 0% or 45%. The higher the volatility, the lower the corresponding expected *geometric* return.

***Exhibit II***



Most diversified portfolios fall in the 9-15% expected standard deviation range (See Exhibit III). Mathematically speaking, any diversification action that reduces portfolio standard deviation from 14% to 10%, but maintains the same *arithmetic* return, would increase *geometric* return by 0.50%.

***Exhibit III***




*For illustrative purposes only to demonstrate the potential differences between arithmetic and geometric returns*

Exhibit IV shows historical *geometric* and *arithmetic* annual returns of the S&P 500 Index from January, 1991 – December, 2005. The *arithmetic* return of the S&P 500 (12.96%) exceeded the *geometric* return (11.52%) by 1.43%, close to the expected difference of 1.63% (calculated based on annual standard deviation).

**Exhibit IV: Arithmetic versus Geometric Returns for the S&P 500 Index**

Arithmetic Return (15 years)

| Calendar Year | US Stocks (S&P 500) |
|---|---|
| 1991 | 30.5% |
| 1992 | 7.6% |
| 1993 | 10.1% |
| 1994 | 1.3% |
| 1995 | 37.6% |
| 1996 | 23.0% |
| 1997 | 33.4% |
| 1998 | 28.6% |
| 1999 | 21.0% |
| 2000 | -9.1% |
| 2001 | -11.9% |
| 2002 | -22.1% |
| 2003 | 28.7% |
| 2004 | 10.9% |
| 2005 | 4.9% |
| Sum of Annual Returns | 194.4% |
| Sum of Annual Returns divided by 15 (years) | 12.96% |
| Average Arithmetic Return | 12.96% |

Geometric Return (15 years)

| Calendar Year | US Stocks (S&P 500) | US Stocks (S&P 500) + 1 |
|---|---|---|
| 1991 | 30.5% | 130.5% |
| 1992 | 7.6% | 107.6% |
| 1993 | 10.1% | 110.1% |
| 1994 | 1.3% | 101.3% |
| 1995 | 37.6% | 137.6% |
| 1996 | 23.0% | 123.0% |
| 1997 | 33.4% | 133.4% |
| 1998 | 28.6% | 128.6% |
| 1999 | 21.0% | 121.0% |
| 2000 | -9.1% | 90.9% |
| 2001 | -11.9% | 88.1% |
| 2002 | -22.1% | 77.9% |
| 2003 | 28.7% | 128.7% |
| 2004 | 10.9% | 110.9% |
| 2005 | 4.9% | 104.9% |
| Product of (annual returns + 1) | | 513.5% |
| Product of (annual returns + 1) to the 1/15 power | | 11.52% |
| Geometric Return | | 11.52% |

| | |
|---|---|
| Annual Standard Deviation | 18.05% |
| Annual Standard Deviation$^2$/2 | 1.63% |
| Arithmetic Return - Geometric Return | 1.43% |

*Past Performance is no guarantee of future returns.*

**The Whole Portfolio's Return Can Be Greater than the Sum of its Parts:**

Exhibit V compares various 15-year return and risk statistics (1991-2005) of *Large Cap US Stocks*, *Commodity Futures*[1] and *Emerging Markets Stocks*. The *Diversified Portfolio* column to the far right reflects statistics for a portfolio that is equally weighted between the three asset classes (and rebalanced annually).

**Exhibit V: Low Volatility Dividend Example**

| Calendar Year | US Stocks (S&P 500) | Commodity Futures (*Dow Jones - AIG Commodity Index) | Emerging Markets (MSCI Emerging Markets) | ***Diversified Portfolio*** *[1/3 US Stocks, 1/3 Commodity Futures and 1/3 Emerging Markets (rebalanced on every 12/31)]* |
|---|---|---|---|---|
| 1991 | 30.5% | -1.1% | 59.9% | 29.7% |
| 1992 | 7.6% | 7.4% | 11.4% | 8.8% |
| 1993 | 10.1% | 5.3% | 74.8% | 30.1% |
| 1994 | 1.3% | 8.6% | -7.3% | 0.9% |
| 1995 | 37.6% | 29.1% | -5.2% | 20.5% |
| 1996 | 23.0% | 21.4% | 6.0% | 16.8% |
| 1997 | 33.4% | -5.8% | -11.6% | 5.3% |
| 1998 | 28.6% | -28.0% | -25.3% | -8.2% |
| 1999 | 21.0% | 21.6% | 66.4% | 36.4% |
| 2000 | -9.1% | 40.7% | -30.6% | 0.3% |
| 2001 | -11.9% | -16.4% | -2.4% | -10.2% |
| 2002 | -22.1% | 44.0% | -6.0% | 5.3% |
| 2003 | 28.7% | 31.9% | 56.3% | 38.9% |
| 2004 | 10.9% | 16.6% | 26.0% | 17.8% |
| 2005 | 4.9% | 21.2% | 34.5% | 20.2% |
| **January 1991 - December 2005 (15 Years)** | | | | |
| Arithmetic Annual Returns | 13.0% | 13.1% | 16.5% | 14.2% |
| Geometric Annual Returns | 11.5% | 11.3% | 11.9% | 13.2% |
| Growth of $1,000,000 | $5,135,369 | $4,958,155 | $5,375,110 | $6,411,648 |
| Annual Standard Deviation | 18.0% | 20.3% | 34.4% | 15.5% |
| St Dev$^2$/ 2 | 1.6% | 2.1% | 5.9% | 1.2% |
| Arith Returns - Geo Returns | 1.4% | 1.8% | 4.6% | 1.0% |

Since *US Stocks, Commodity Futures* and *Emerging Markets* had relatively low correlation to each other during the 15-year sample period; the *diversified portfolio* (made up of all three) had lower volatility than each asset class by itself. The *diversified portfolio*, rebalanced annually, would have generated a 13.2% annualized geometric return. Meanwhile, the average return of *US Stocks(11.5%), Commodity Futures(11.3%)* and *Emerging Markets(11.9%)* was 11.6%. Therefore, the diversified mix would have generated 1.6% more return per year. So in effect, the *diversified portfolio* as a whole generated a greater return than the sum of its parts. Since the diversification of the three asset classes tightened the band between high and low returns, the volatility (standard deviation) declined. The lower the volatility, the less "damage" is done to the *geometric* return. Or said another way, the lower the volatility, the closer the arithmetic and *geometric* returns become.

*[1] Assumes DJ-AIG Commodity Index was collateralized with Citigroup Infl-linked Sec index (97-04) & Lehman Agg. Bond index (91-97).*
*Past performance including risk, return and correlation is no guarantee of future performance including risk, return and correlation.*
*Investments in Commodity futures, US Stocks and Emerging markets can be very risky. This analysis does not constitute an investment recommendation.*

**Conclusion:**

As shown in Exhibit V, the *diversified portfolio* generated more than $1,000,000 in excess returns than what the highest returning asset class (*Emerging Markets*) would have generated by itself. While *Commodity Futures* and *US Stocks* had lower returns than *Emerging Markets* over the last 15-year time-period in question, they still added to the *geometric* return when combined with *Emerging Markets* within the *diversified portfolio*.

The *arithmetic* annual return of a portfolio is simply the weighted average annual *arithmetic* return of its underlying investments[2]. However, if the underlying investments in a portfolio are not perfectly positively correlated, the *geometric* return of a portfolio is greater than the weighted average *geometric* return of the portfolio's underlying investments[3]. The traditional Markowitz mean-variance optimization model uses *arithmetic* return input assumptions (along with standard deviation & correlation) to optimize for overall portfolio *arithmetic* return and risk. In Exhibit V, *Emerging Markets* had the highest 15-year *arithmetic* return and would have warranted a 100% allocation if the goal were to maximize *arithmetic* returns. So while helpful in an academic setting to illustrate some of the benefits of diversification, clearly such output from the traditional Markowitz optimization model has little practical value. For real-world applications, portfolio optimization models should optimize for time-horizon *geometric* returns.

The **Low Volatility Dividend** could lead one to conclude that broad diversification among many low correlating assets is essential to not only control risk, but to maximize long-term returns. Such non-traditional and low-correlating asset classes that may warrant shelf space in a diversified portfolio include *inflation-indexed bonds*, *foreign bonds*, *high yield bonds*, *REITS*, *Foreign Real Estate, Emerging Markets Equity*, *Emerging Market Debt*, *Commodity Futures*, *Timberland*, *Hedge Funds (absolute return strategies)*, among others.

*Note: DiMeo Schneider & Associates, LLC's proprietary probabilistic asset allocation model, The Frontier Engineer™, optimizes for expected geometric returns. Please contact your consultant or Matthew Rice for more information at 312-853-1000.*

***About the Author:***

*Matthew Rice, CFA, CIMA: Matt is the firm's Chief Research Officer and a Senior Consultant. As Chief Research Officer, Matt spearheads the firm's research efforts in the areas of capital market analysis & forecasts, investment strategy, asset allocation & portfolio rebalancing modeling and alternative investments. He is also a member of the DiMeo Schneider & Associates, L.L.C.'s investment committee which sets investment policy, establishes the framework for asset allocation and approves investment managers. In 2004, he co-authored The Practical Guide to Managing Nonprofit Assets (John Wiley & Sons). Matt received a BA in Economics from Northwestern University, is CFA Charterholder (Chartered Financial Analyst), a CIMA (Certified Investment Management Analyst), a CIMC (Certified Investment Management Consultant), and earned an Alternative Investment Certificate from the Wharton School of Business & IMCA. Prior to joining DiMeo Schneider & Associates, L.L.C., Matt was a Trust Officer in the institutional investment services group at Fifth Third Bank (formerly Old Kent Bank), worked as a Corporate Retirement Plan Consultant for First Business Investment Services, and was an Investment Consultant at AXA Advisors. Prior to beginning his investment career, he signed a free-agent NFL contract with the Arizona Cardinals.*



[2] *Assuming annual rebalance*
[3] *Assuming annual rebalance*